A Group Theoretical Analysis Of Constituent Gluons In Scalar Glueballs

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Abstract

Young Tableau in Group theory is a method of determining number of irreducible representations of direct products in SU(N) groups, while glueballs are predicted hadronic states in QCD consisting only of Gluons without quarks and antiquarks. We present results of our work using it in determining the number of constituent gluons in low lying scalar glueballs.

1 Introduction

Quantum Chromodynamics (QCD) is the theory of the hadronic interactions. In QCD, the gluons are massless and have spin-1. Gluons themselves carry color charge, and so they can interact with other gluons [1]. In particle Physics, a glueball is a hypothetical composite particle. It consists solely of gluons, without valence quarks. Such a state is possible because gluons carry color charge and experience the strong interaction. Glueballs have not been an easy subject to study due to the lack of phenomenological support and therefore much debate has been associated with their properties. Glueballs are extremely difficult to identify in particle accelerators, because they mix with ordinary meson states. Glueball is mainly produced in radiative J/ψ decay and $p\bar{p}$ annihilation. Theoretical calculations based on nonperturbative methods like Lattice QCD and QCD sum rules agree that the lightest glueball should be a scalar resonance ($J^{pc} = 0^{++}$) with a mass range 1400-1800 MeV^6 followed by a tensor (2^{++}) and a pseudoscalar (0^{-+}) glueball in the 2000 -2500 MeV mass region [2]. So far,there has been 20 years of intensive experimental search towards glueballs. Unfortunately, no definite answer to the question whether a glueball has been observed or not can be given. It is to be mentioned that although gluons are color octet, glueballs are color singlet. So a single gluon cannot be a glueball, but a gluelump [3]. The forthcoming experiment FAIR has PANDA as detector [4], specifically designed to detect glueballs, hybrid mesons and charmonium spectroscopy. Theoretical calculations show that glueballs should exist at energy ranges accessible with current collider technology. However, due to the aforementioned difficulty, they have so far not been observed and identified with certainty. But still it is a worthwhile theoretical pursuit.

In the present paper, we report the results for scalar glueballs using Young Tableau. In the previous publications [2,5], we have presented the corresponding results for pentaquark multiplets.

In section 2, we outline the formalism; section 3 the results; section 4 the experimental status of glueballs; whether section 5 is devoted to conclusion.

2 Formalism

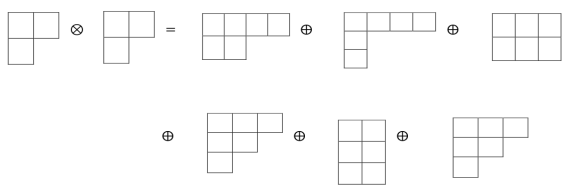
In this section we outline the method how group theoretical tool of Young Tableau of $SU(3)_c$ can be used to find the maximum number of constituent gluons in an experimentally observed scalar glueball.

In Mathematics, a Young tableau is a combinational object useful in representation theory. Young tableaux were introduced by Alfred Young, a mathematician at Cambridge University, in 1900. By taking the direct product of irreducible representations we can generate the representations of higher dimensions. These representations are however reducible. Young tableau gives a definite way of reducing it to direct sum of various irreducible representations [6,7]. The theory in detail is discussed in ref. [6,7].

However this tool can only be used for low lying scalar C= +1 glueballs. Because only for scalar Glueball l=0 and s=0; J=0. Similarly in case of scalar Glueball there will not be any additional angular momentum multiplicity.

3 Results

Here we show the product result for $(8_c \times 8_c)$ in $SU(3)_c$:



$$= 27 + 10 + \bar{10} + 8 + 1 + 8$$

Figure 1: Results for glueball consist of two Gluons

This shows two color-octet gluons can form a color singlet. Thus if only one glueball is observed, the glueball must be composed two gluons. The lightest scalar glueball mass is in the range of 1500-1750 MeV [8]. Similarly, from the bound state of three or more gluons the corresponding representations can be found out (Table-1).

Table-1 shows the number of color singlet glueballs and the number of constituent gluons:

Table 1: Number of color singlets corresponding to the maximum no. of constituent gluons

Direct product	Direct sum	Number of Color Singlet
$8_c \times 8_c$	$\underline{1}+\underline{8}+\underline{8}+\underline{10}+\underline{\overline{10}}+\underline{27}$	1
$8_c \times 8_c \times 8_c$	$2(\underline{1}) + 8(\underline{8}) + 4(\underline{10}) + 4(\underline{\overline{10}}) + 6(\underline{\bar{2}7}) + 4(\underline{35}) + \underline{64}$	2
$8_c \times 8_c \times 8_c \times 8_c$	$8(\underline{1}) + 32(\underline{8}) + 22(\underline{10}) + 18(\underline{\overline{10}}) + 33(\underline{27}) + 4(\underline{28}) + 30(\underline{35}) + 12(\underline{64}) + 6(\underline{81}) + \underline{125}$	8
$8_c \times 8_c \times 8_c \times 8_c \times 8_c$	$32(\underline{1}) + 145(\underline{8}) + 117(\underline{10}) + 83(\underline{\overline{10}}) + 180(\underline{27}) + 40(\underline{28}) + 200(\underline{35}) + 94(\underline{64}) + 10(\underline{80}) + 72(\underline{81}) + 20(\underline{125}) + 8(\underline{154}) + \underline{216}$	32
$8_c \times 8_c \times 8_c \times 8_c \times 8_c \times 8_c$	$145(\underline{1}) + 702(\underline{8}) + 642(\underline{10}) + 408(\underline{\overline{10}}) + 999(\underline{27}) + 322(\underline{28}) + 1260(\underline{35}) + 10(\underline{55}) + 660(\underline{64}) + 140(\underline{80}) + 630(\underline{81}) + 215(\underline{125}) + 140(\underline{154}) + 18(\underline{162}) + 30(\underline{216}) + 10(\underline{260}) + \underline{343}$	145
$8_c \times 8_c \times 8_c \times 8_c \times 8_c \times 8_c \times 8_c$	$702(\underline{1}) + 3598(\underline{8}) + 3603(\underline{10}) + 2109(\underline{\overline{10}}) + 5670(\underline{27}) + 7840(\underline{35}) + 2352(\underline{28}) + 168(\underline{55}) + 4424(\underline{64}) + 1400(\underline{80}) + 4872(\underline{81}) + 1890(\underline{125}) + 1568(\underline{154}) + 336(\underline{162}) + 426(\underline{216}) + 239(\underline{260}) + 41(\underline{343}) + 28(\underline{280}) + 12(\underline{405}) + \underline{512} + \underline{273} + \underline{330}$	702

It shows that the Young Tableau calculation gives us the possibility to infer the maximum number of constituent gluons from the given number of experimentally observed scalar glueballs, as shown in table-2 upto multiplicity 702.

Table-2 shows maximum number of constituent gluons in such Glueball muliplicity.

Table 2: Possible number of constituent gluons corresponding to the number of scalar glueballs observed

No. of scalar glueballs observed	Possible No. of constituent gluons
1	2
2-7	2,3
8-31	2,3,4
32-144	2,3,4,5
145-701	2,3,4,5,6
702	2,3,4,5,6,7

4 Experimental status of Glueballs

Good evidence exists for a scalar glueball which is mixed with nereby mesons, but a full understanding is still missing. Evidence for tensor and pseudoscalar glueballs are weak at best [9]. Glueballs should, e.g., be produced preferentially in so-called gluon-rich processes [9,10]:

(i) **Radiative J/ψ decay**: In the most decays, the J/ψ undergoes a transition into 3 gluons which then convert into hadrons. But the J/ψ can also decay into 2 gluons and a photon

$$\mathrm{J}/\psi \longrightarrow \gamma gg \longrightarrow \gamma G \qquad [9]$$

The photons can be detected, the two gluons interact and must form glueballs - if they exists [10].

(ii) $p\bar{p}$ **annihilation**: In $p\bar{p}$ annihilation, quark-antiquark pairs annihilate into gluons, they interact and may form glueballs [10].

(iii) **Central production**: In central production two hadrons pass by each other 'nearly untouched' and are scattered diffractively in forward direction. The valence quarks are exchanged. The process is often called Pomeron-Pomeron scattering. The absence of valence quarks in the production process makes central production a good place to search for glueballs [10].

There are many new experiments planned, e.g. the PANDA Experiment at GSI in Germany [11], BES III at BEPC II in Bejing [12], the GlueX Experiment at Jefferson Laboratory in the USA [13], ALICE at CERN [14][15] which will provide us more data on this. On the one hand, some experimental glueball candidates are currently known. Most of them are scalar, such as the $a_0(980)$, $f_0(980)$, $f_0(1500)$, $f_0(1710), ...$ but no definitive conclusions can be drawn concerning the nature of these states [16] nor its exact multiplicity.

5 Conclusion

We have shown how one can use Young Tableau to infer the number of costituent gluons from the multiplicity of low lying glueballs. In lattice QCD calculation, low lying C= +1 glueballs are identified with two gluon states (or at least with hadrons in which two gluon components widely dominent), while no light C=-1 (l+s= odd) and heavy scalar $J^{pc} = 0^{+-}$ glueballs are seen as light four gluon states. The group theoretical approach cannot make distinction among the glueballs of different number of constituent gluons. Only constraint is that the glueballs must have total orbital momentum and total spins of the n constituent gluons be seperately 0, so that angular momentum does not have role in its spectroscopy. The method however falls short of accomodating gluelump [3] having single constituent gluon with additional gluon field.

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